Chiral vortons and cosmological constraints on particle physics

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We investigate the cosmological consequences of particle physics theories that admit stable loops of current-carrying string—vortons. In particular, we consider chiral theories where a single fermion zero mode is excited in the string core, such as those arising in supersymmetric theories with a D term. The resulting vortons formed in such theories are expected to be more stable than their nonchiral cousins. General symmetry breaking schemes are considered in which strings formed at one symmetry breaking scale become current carrying at a subsequent phase transition. The vorton abundance is estimated and constraints placed on the underlying particle physics theories from cosmological observations. Our constraints on the chiral theory are considerably more stringent than the previous estimates for more general theories.

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I. INTRODUCTION

There is considerable interest in supersymmetric theories, both from a theoretical and phenomenological viewpoint. Indeed, current experiments seem to suggest that the inclusion of supersymmetry is necessary for the couplings to unify in a grand unified theory and to solve the hierarchy problem. Supersymmetry also underpins superstring theory. It is thus important to explore the cosmological consequences of such theories.

Recently it was shown that many supersymmetric theories admit cosmic strings [1,2]. The microphysics of such strings were investigated and it was shown that they necessarily result in fermion zero modes in the string core. The importance of this result is that the fermion zero modes can be excited and move along the string. This completely changes its properties, resulting in it becoming a current-carrying string [6]. Both Abelian [1] and non-Abelian [2] theories were considered, and similar results found in both cases. For the Abelian case symmetry breaking by both an F term and a D term were considered. For the theory with an F term it was found that there is a pair of fermion zero modes, one left mover and one right mover. However, in the D-term case there was a single fermion zero mode, either a left mover or a right mover.

Since supersymmetry is not observed in nature the effect of supersymmetry breaking was also considered [2]. In most cases supersymmetry breaking enables the left and right movers to mix, destroying the zero modes, However, for the D-term theory the chiral zero mode survives supersymmetry breaking because there is no other zero mode for it to mix with. This theory is of importance because many superstring compactifications result in a $U(1)$ symmetry being broken with a D term, hence giving rise to the cosmic strings considered in [1]. However, the presence of fermion zero modes completely changes the cosmology of such strings and could render the theory cosmologically unacceptable. The reason for this bold statement is the following. An initially weak current on a string loop is amplified as the loop contracts. The current becomes sufficiently strong to halt the loop contraction, preventing it from decaying. A stable state, or vorton [3], is formed. Chiral vortons belong to the category that is classically stable [4], though their quantum mechanical stability is an open question. The density of vortons is tightly constrained by cosmological requirements. For example, if vortons are sufficiently stable to survive until the present time, then their density must be such that the universe does not become vorton dominated. On a more conservative note, if the vortons only live a few minutes, then their density must be such that the universe is radiation dominated at nucleosynthesis. This enables us to constrain theories that give rise to current-carrying strings.

While [1,2] showed that all supersymmetric theories result in current-carrying strings, supersymmetry breaking could destroy the zero modes in the F-term theories. Thus, the vorton problem could solve itself here. However, for Abelian theories with symmetry breaking implemented with a D term, the zero mode survives, resulting in a potential vorton problem. In a previous paper we constrained particle physics theories giving rise to current-carrying strings [5]. The constraints applied to cases where the string became current carrying at formation or else at a subsequent phase transition, and led to fairly stringent bounds on current-carrying strings. However, the problem in the chiral theory is more serious than the theories previously considered. This is because we previously considered a random current on the cosmic string. However, in the chiral case, the fermion zero mode can travel in one direction only, resulting in a maximal current and consequently a maximum vorton density.

In this paper we revise our previous estimates for this chiral case. Since these cosmic strings arise in a wide class of supersymmetric and superstring theories our results are potentially important.

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II. COSMIC VORTONS

A. The case of strictly chiral vorton states

In a previous paper [5] we considered the generic case of cosmological vorton production, which is characterized by two independent quantum numbers, N and Z. The circumference of the vorton is given approximately by

$$l_v \simeq (2\pi)^{1/2}|NZ|^{1/2} m_x^{-1}, \tag{1}$$

where m_x is the relevant Kibble mass scale, which is defined as the square root of the string tension $\mathcal{T}$ in the low current limit. This results in the vorton energy,

$$E_v \simeq l_v m_x^2. \tag{2}$$

Since even in the general case one would expect the two quantum numbers to have comparable magnitude, the typical vorton mass would be

$$E_v \approx N m_x. \tag{3}$$

Here we will consider the more specific case where the current is strictly chiral, so that only one of the numbers N and Z can be chosen independently, while the other will be given by the exact equality

$$Z = N. \tag{4}$$

As in the earlier work [5], the present work is carried out within a scheme depending on two basic mass scales characterizing the temperatures of two distinct condensation processes. The first is the temperature,

$$T_x \approx m_x, \tag{5}$$

at which the strings themselves are formed. The second is the comparable or lower temperature given by

$$T_\sigma \approx m_\sigma, \tag{6}$$

where m_σ is the relevant carrier mass scale characterizing a current that condenses on the string in the manner described by Witten [6].

In the strictly chiral case [7], which will be our main concern, the current can only be fermionic and must be electromagnetically decoupled. However, whether strictly chiral or otherwise, the requirement that the current be able to provide the centrifugal support for an effectively stable classical vorton state requires that the relevant vorton circumference l_v should be large compared with the Compton wavelength associated with the carrier mass m_σ. This requirement, namely,

$$l_v \gg m_\sigma^{-1}, \tag{7}$$

will only be satisfied if

$$|NZ|^{1/2} \gg \frac{T_x}{T_\sigma}. \tag{8}$$

A loop that failed to satisfy this requirement would not be destined to survive as a vorton.

B. The underlying cosmological scenario

Like its predecessor [5], the present analysis will be carried out within the framework of a simple big bang theory of the usual type in which the universe evolves in approximate thermal equilibrium with a cosmological background temperature T. The effective massless degrees of freedom at temperature T is denoted by g^*, where $g^* \approx 1$ at low temperatures but that in the range where vorton production is likely to occur, from electroweak unification through to grand unification, something like $g^* \cong 10^2$ is a reasonable estimate.

All likely vorton formation processes occurred during the radiation-dominated era and ended when the universe dropped below the Rydberg energy scale, of the order of 10^{-8} GeV and became effectively transparent. According to the Friedmann formula, the age t of the universe is given by $t \approx H^{-1}$, and H is the Hubble parameter. During the radiation-dominated era, the cosmological time is given by

$$t \approx \frac{m_P}{\sqrt{g^*}T^2}, \tag{9}$$

where $m_P = G^{-1/2}$ is the Planck mass.

As discussed in the following sections, any string loops present at the epoch of the current condensation process at the temperature given by (6) will be endowed by the ambient thermal fluctuations with corresponding quantum numbers N and Z that will necessarily be equal in the chiral case, and that subsequently, at lower temperatures, will be conserved in between loop chopping and recombination processes unless or until the loop length gets below the quantum stability limit (7). In the long run the loops with quantum numbers large enough for the corresponding stability requirement (8) to be satisfied will be predestined to become stationary vortons; we refer to these as *protovortons*.

In the strictly chiral case the relevant quantum numbers can be expected to be considerably larger than in the generic case considered before. This implies that, as in the generic case, so *a fortiori* in the chiral case, whenever the carrier condensation occurs during the friction-dominated regime, a majority of the loops already present at the temperature T_σ of the carrier condensation will qualify as protovortons. However, in other cases (those for which the condensation occurs at a relatively low temperature) it will only be at a later stage that the protovorton loops get sufficiently free from the surrounding tangle of string for subsequent quantum number changing intersections to be negligible. The correspondingly lower temperature, $T_f \lesssim T_\sigma$, at which this occurs will be referred to as the protovorton formation temperature. The protovortons will not become vortons in the strict sense until what may be an even lower temperature, the vorton relaxation temperature T_r, say (whose value will not be relevant for our present purpose) since the loops must first lose their excess energy. Whereas frictional drag and particle production will commonly ensure fairly rapid relaxation, there may be cases in which the only losses are due to the much weaker mechanism of gravitational radiation.

The raw material for vorton production is provided by the process whereby, as the string distribution rarifies due to friction and radiation damping, not all of its lost energy goes directly into frictional heating of the background or emitted radiation. Instead there will always be a certain fraction ε, say, that goes into autonomous loops, meaning loops small enough to evolve without subsequent collisions with the main string distribution. Since these loops cannot greatly exceed the smoothing length scale they will not have much fine substructure and their subsequent evolution is very likely to satisfy the condition of avoiding quantum number changing self-intersections as they subsequently contract.

When the condensation occurs during the friction-dominated epoch, the autonomous loops that emerge at the condensation temperature T_σ will satisfy the condition (8) and thus be describable as protovortons. However, for lower values of T_σ the majority of the loops that emerge during the period immediately following the carrier condensation will be too small to have aquired sufficiently large quantum numbers, and therefore will not be viable in the long run. Nevertheless, even in such unfavorable circumstances, the monotonic increase of the damping length scale ξ will ensure that at some lower temperature, $T_f < T_\sigma$, a later—but therefore less prolific—generation of emerging loops will after all be able to qualify as protovortons.

The small autonomously evolving loops that we refer to as protovortons are supposed to be sufficiently small compared with the ever expanding length scales characterizing the rest of the string distribution and sufficiently smooth (due to previous damping) to avoid destructive fragmentation by self-collisions. Thus in most individual cases, with reasonably high accuracy when averaged, the relevant quantum numbers N and Z will be conserved. As a consequence, the statistical properties of the future vorton population will be predetermined by those of the corresponding protovorton loops at the time of their emergence at the temperature T_f.

C. The smoothing length scale

The scenario summarized above is based on the accepted understanding of the Kibble mechanism [8], according to which, after the temperature has dropped below T_x the effect of various damping mechanisms will remove most of the structure below an effective smoothing length ξ which will increase monotonically as a function of time, so that nearly all the surviving loops will have a length $L = \oint dl$ that satisfies the inequality

$$L \gtrsim \xi. \tag{10}$$

There will thus be a distribution of string loops, of which the most numerous will be relatively short ones, with $L \approx \xi$, that are on the verge of emerging, or that have already emerged, as autonomous protovortons or as loops that are about to contract and disappear, as the case may be. Above this smoothing length there will be a spectrum of tangled structure, in which the number density n, say, of closed loops and wiggles segments extending over a radial distance greater than $R > \xi$ will be given by an expression of the form

$$n \approx \frac{\nu}{R^3}, \tag{11}$$

in which ν is a dimensionless coefficient. Note that an individual closed loop characterized by a radial extension of order R will typically have a much longer total random walk length given by

$$L \approx R^2/\xi. \tag{12}$$

When the string distribution is first formed, at the temperature T_x, one expects [8] that the spectrum will be of the simple Brownian type for which ν has a constant value ν_* say of order unity. By causality the spectrum must always retain a simple Brownian form for values of R exceeding the horizon length scale t. The Brownian spectrum will be preserved even in the intermediate range $\xi < R < t$ throughout the friction-dominated regime. However, as discussed below, the situation becomes more complicated in the subsequent regime of locally free string motion whose description requires the use of a variable coefficient ν that will depend on both R and t.

Whereas on larger scales closed loops and wiggles on very long string segments will be tangled together, on the shortest scales characterized by the lower cutoff ξ, loops of a relatively smooth form lead a comparatively autonomous existence, passing between the meshes of the ambient tangle with only occasional collisions. It is these smallest loops that are of interest as candidates for subsequent transformation into vortons. If one adopts the convention that these autonomous loops are to be counted apart from the main string distribution, the extrapolation of the spectrum (11) to wavelengths small compared with ξ will be describable by a drastic reduction of ν to a negligibly small value in this range, so that the overall spectrum (11) will peak at the value $R \approx \xi$. Alternatively, if one adopts the convention that not only wiggles but also the autonomous small loops are to be included in the count, then there will still be a reduction on ν, but of a more moderate nature, so that instead of a peak the overall spectrum will just have a plateau for $R \lesssim \xi$.

The total number density of the small autonomous loops with length and radial extension of the order of ξ will (due to the rapid falloff of the spectrum that is expected for larger scales) be not much less than the number density of all closed loops (including those that exist ephemerally pending string intersections at larger scales), and so will be given by an expression of the form

$$n \approx \overline{\nu} \xi^{-3}, \tag{13}$$

where $\overline{\nu}$ is the value of ν for values of R of the order of ξ. In view of the expected falloff of the spectrum, this value $\overline{\nu}$ will also be interpretable as an appropriately averaged value of ν. In particular, for the Brownian case in which ν has a constant value ν_* over the range $R \lesssim \xi$, we shall evidently have $\overline{\nu} \approx \nu_*$.

III. THE VORTON POPULATION

A. The chiral amplification effect

The standard theory reviewed above was originally developed on the assumption that the string evolution is governed by Nambu-Goto-type dynamics. However, this will change after the current carrier condensation, at the temperature T_σ. The reason why the strings not only can but will carry a potentially significant current is that the typical length scale ξ of the string loops at the transition temperature T_σ will generally have a value, ξ_σ, say, that is considerably greater than the relevant value $\lambda_\sigma \approx T_\sigma^{-1}$ of the wavelength λ characterizing the local current fluctuations induced on the string by the thermal background.

In the generic case studied previously [5] the number $N \approx L/\lambda$ of such fluctuation lengths around any loop of circumference $L \gtrsim \xi$ will overestimate the corresponding total quantum number associated with the loop, which by a ''random walk'' argument can be expected to be only of the order of the corresponding square root, namely, $N \approx \sqrt{L/\lambda}$. However, in the strictly chiral case, in which only right (or left) moving null modes are allowed, the partially cancelling backwards steps in the ''walk'' are not allowed, so the naive estimate $N \approx L/\lambda$ will be valid. Both possibilities can be allowed for simultaneously by using the formula

$$|Z| \approx N \approx \left(\frac{L}{\lambda}\right)^{1/i}, \tag{14}$$

with $i=1$ for the strictly chiral case, and $i=2$ for the generic case that was studied previously.

Since the loop length L is bounded below by a smoothing length ξ that will be large compared with the relevant fluctuation wavelength

$$\xi_\sigma \gg \lambda_\sigma, \tag{15}$$

when the current first condenses, it can be seen that even in the generic case $i=2$, and *a fortiori* in the strictly chiral case $i=1$, the quantum numbers provided by the formula (14) will be large compared with unity. A typical loop at that time will be characterized by

$$L \approx \xi_\sigma \tag{16}$$

and

$$\lambda \approx T_\sigma^{-1}, \tag{17}$$

so that one obtains the estimate

$$|Z| \approx N \approx (\xi_\sigma T_\sigma)^{1/i}. \tag{18}$$

For current condensation during the friction-dominated regime this will always be enough to fulfill the requirement (8), but this condition, namely,

$$\xi_\sigma \gg \frac{m_x^i}{T_\sigma^{i+1}}, \tag{19}$$

will not hold for condensation later on in the radiation damping regime. In the latter case, typical small loops that free themselves from the main string distribution at or soon after the time of current condensation will ultimately disappear, since they acquire their autonomy too soon to be viable in the long run. However, there will always be a minority of longer loops, namely, those exceeding a minimum given by

$$L \approx \frac{m_x^i}{T_\sigma^{i+1}}, \tag{20}$$

for which (8) will be satisfied.

The condition (20) is still not quite sufficient to qualify all of these loops as protovortons since such exceptionally long loops will be very wiggly and collision prone. In such cases it is not until a later time at a lower temperature T_f that free protovorton loops will emerge. By this stage, instead of its original value (17), the typical wavelength of the carrier field will be given by an expression of the form

$$\lambda \approx \mathcal{Z}_f T_\sigma^{-1}, \tag{21}$$

which involves a blueshift factor $\mathcal{Z}_f$ whose value is not immediately obvious but is needed to allow for the net effect on the string of weak stretching due to the cosmological expansion and stronger shrinking due to wiggle damping during the intervening period as the temperature cools from T_σ to T_f.

In the earlier friction-dominated regime T_f is identifiable with T_σ so the problem does not arise, and trivially $\mathcal{Z}_f = 1$. In the radiation damping era, for which the evaluation of $\mathcal{Z}_f$ is needed, cosmological stretching will in fact be negligible, so obviously the net effect is that $\mathcal{Z}_f$ will be small compared with unity. The hard part of the problem is to estimate by how much so. It is also obvious that since, during the same period, the smoothing length ξ can only increase, the corresponding final value

$$L \approx \xi_f \tag{22}$$

of the length of a typical loop cannot be less than the original value (16). It therefore follows that the new estimate

$$|Z| \approx N \approx \left(\frac{\xi_f T_\sigma}{\mathcal{Z}_f}\right)^{1/i}, \tag{23}$$

obtained from (14) will certainly be larger than the previous value (18). More specifically, the value given by (23) will increase monotonically as the value of the final temperature T_f for which it is evaluated diminishes.

The required value of T_f—at which the formation of the protovorton loops occurs—is that for which the monotonic function (23) reaches the minimum qualifying value given by (8). This value is thus obtainable in principle by solving the equation

$$\frac{\xi_f}{\mathcal{Z}_f} \approx \frac{m_x^i}{T_\sigma^{i+1}}, \tag{24}$$

but this can only be done in practice when we have found the dependence on T_f not only of ξ_f, which is comparatively easy, but also of $\mathcal{Z}_f$ which is rather more difficult.

The number density of the protovorton loops when they are first formed at the temperature T_f will be comparable with the total loop number density at the time. By (13) it will be expressible as

$$n_f \approx \varepsilon \nu_f \xi_f^{-3}, \tag{25}$$

where ε is an efficiency factor of order unity, and where ν_f is the value of the dimensionless parameter $\overline{\nu}$ at that time. This will also simply have an order of unity value, $\nu_f \approx \nu_\star$, say, if the current condenses in the friction-dominated regime, for which $T_f \approx T_\sigma$. However, ν_f can be expected to have a lower value if the condensation does not occur until later on in the radiation-dominated era.

In all cases, since the number of protovorton loops in a comoving volume will be approximately conserved during their subsequent damping-dominated evolution, then the number density n_v of the resulting vortons later on at a lower temperature T will be given in terms of the number density n_f of the protovorton loops at the time of condensation by

$$\frac{n_v}{n_f} \approx \frac{f}{\varepsilon}\left(\frac{T}{T_f}\right)^3, \tag{26}$$

where f is a dimensionless adjustment factor that we expect to be small but not very small compared with unity, and that will be given by

$$f \simeq \frac{\varepsilon g^*}{g_f^*}, \tag{27}$$

where g_f^* is the value of g^* at the protovorton formation temperature T_f.

It follows from (3) that the corresponding mass density will be given by

$$\rho_v \approx N m_x n_v . \tag{28}$$

Using the preceding estimates, the temperature dependence of the mass density is found to be given by the general formula

$$\rho_v \approx f \nu_f \frac{m_x T_\sigma^{1/i}}{\mathcal{Z}_f^{1/i} \xi_f^{(3i-1)/i}} \left(\frac{T}{T_f}\right)^3, \tag{29}$$

in which we shall simply have $T_f \approx T_\sigma$ and therefore $\mathcal{Z}_f \approx 1$ whenever (8) is satisfied by (18). It is now necessary to evaluate this in the friction and radiation damping regimes.

B. Condensation in the friction damping regime

If the current condensation occurs during the friction-dominated epoch, the evaluation of the quantities involved in (29) is comparatively straightforward. According to the standard picture [9], as the background temperature T drops below the string formation temperature T_x of the relevant symmetry breaking phase transition, the evolution of the cosmic string network will at first be dominated by the frictional drag of the thermal background. It has been predicted that the relevant dynamical damping time scale τ during this period will be approximately given by

$$\tau \approx \frac{T_x^2}{\beta T^3}, \tag{30}$$

where β is a dimensionless drag coefficient that depends on the details of the underlying field theory, but that is typically expected [9,10] to be roughly of the order of unity, $\beta \approx 1$. The effect of the damping is to freeze the large scale structure, so that it retains the Brownian random walk form described by (11) with a fixed order of unity value $\nu_\star$, say, for the dimensionless coefficient ν, while it smooths out the microstructure below a correlation length scale ξ given by

$$\xi \approx \sqrt{\tau t}, \tag{31}$$

where t is the Hubble time scale, which is given by (9). The required correlation length scale is thus finally found to be given in order of magnitude by

$$\xi \approx \left(\frac{m_P}{\beta}\right)^{1/2} \frac{T_x}{T^{5/2}} \tag{32}$$

(neglecting the very weak g^* dependence that would be contained in a factor $g^{*1/4}$, on the assumption that this factor will not be far from unity).

The validity of the above derivation is of course based on the supposition that the cosmological time scale t is longer than τ, so that the damping process can actually be effective. It can be seen from the preceding formulas for t and τ that this condition will indeed be satisfied just as long as the temperature T remains above a critical value $T_\star$ given by

$$T_\star \approx \frac{T_x^2}{\beta m_P}. \tag{33}$$

So long as the condensation temperature exceeds this critical value, i.e., provided it lies in the friction-drag-dominated regime

$$T_x \gtrsim T \gtrsim T_\star , \tag{34}$$

it can be verified that the majority of the small string loops already present at the time will satisfy the condition (8) for qualification as protovortons, which means that the appropriate value of L for substitution in (18) will be given by (16). It can be thus seen from (25) and (32) that the number density of these protovorton loops will be given at the time of their formation by

$$n_f \approx \varepsilon \nu_\star \left(\frac{\beta T_\sigma}{m_P}\right)^{3/2} \left(\frac{T_\sigma^2}{T_x}\right)^3, \tag{35}$$

where $\nu_\star$ is the constant order of unity value that is retained, during the friction-dominated era, by the coefficient $\overline{\nu}$ in (13) and hence by ν_f in (25). It follows from (26) that at later times the number density of their mature vorton successors will be given in order of magnitude by

$$n_v \approx \nu_\star f\left(\frac{\beta T_\sigma}{m_P}\right)^{3/2}\left(\frac{T_\sigma T}{T_x}\right)^3. \quad (36)$$

Thus, after the temperature has fallen below the value T_r the order of magnitude of the resulting mass density ρ_v of the relic vorton population, to which the cosmic string loop distribution will have been reduced, will be

$$\rho_v \approx \nu_\star f N\left(\frac{\beta T_\sigma}{m_P}\right)^{3/2}\left(\frac{T_\sigma}{T_x}\right)^2 T_\sigma T^3. \quad (37)$$

Setting T equal to T_σ in (32) in order to obtain the relevant value of ξ_σ, and using (18), the required expectation value for the quantum number N in the previous formula can be estimated as

$$N \approx \left(\left(\frac{m_P}{\beta T_\sigma}\right)^{1/2}\frac{m_x}{T_\sigma}\right)^{1/i}. \quad (38)$$

It follows from (3) and (1) that a typical vorton in this relic distribution will have a mass energy given by

$$E_v \approx \left(\frac{m_P}{\beta T_\sigma}\right)^{1/2i}\left(\frac{m_x}{T_\sigma}\right)^{1/i} m_x. \quad (39)$$

It can thus be confirmed using (33) that, as stated above, the postulate $T_\sigma > T_\star$ automatically ensures that these vortons will indeed satisfy the minimum length requirement (7) by a considerable margin in the strictly chiral case, though only marginally when T is at the lower end of this range.

For the resulting distribution of vortons, the mass density obtained by substituting (38) in (37) is found to be given by

$$\rho_v \approx \nu_\star f\left(\frac{\beta T_\sigma}{m_P}\right)^{3/2-1/2i}\left(\frac{T_\sigma}{m_x}\right)^{2-1/i} T_\sigma T^3. \quad (40)$$

In the strictly chiral case $i=1$, this result will be expressible by

$$\frac{\rho_v}{T^3} \approx \frac{\nu_\star f \beta T_\sigma^3}{m_P m_x}. \quad (41)$$

It is to be remarked that unlike its absolute value, the relative augmentation factor of the mass density does not depend on the as yet rather uncertain efficiency factor ε (which seems likely to be very small in the friction-dominated epoch, but perhaps nearer the order of unity in the radiation-reaction-dominated epoch). It can be seen that in comparison with the previously studied generic case, $i=2$, the mass density is augmented for the strictly chiral case, $i=1$, by a factor that is simply expressible as $(m_x/T_\sigma)^{1/2}(m_P/\beta T_\sigma)^{1/4}$.

C. Condensation in the radiation damping regime

For strings that may have been formed in some (nonstandard, e.g., supersymmetric) electroweak symmetry breaking transition, the scenario of the preceding subsection is the only one that needs to be considered. However, for strings formed at much higher energies, in particular for the commonly considered case of grand unified theory (GUT) strings, there will be an extensive temperature range below the Kibble transition value $T_\star$ at which friction becomes unimportant during which the current condensation could ocur. Since we saw that the minimum length requirement was only marginally satisfied by typical loops when the condensation occurred near the end of the friction-dominated regime, it is to be expected that in the cases to be considered here for which the condensation temperature T_σ occurs below the transition temperature $T_\star$ given by (33), typical loops present during the transition will not be long enough to qualify as protovortons. This means that the vorton formation temperature T_f will not coincide with T_σ as it did in the friction-dominated regime, but that it will have a distinctly lower value, so the scenarios to be considered in this section will be characterized by

$$T_\star > T_\sigma > T_f. \quad (42)$$

In such a scenario the final stage of protovorton formation will be preceeded by a period of evolution in the temperature range $T_\star \gtrsim T \gtrsim T_f$ during which the effect of friction will be negligible, so that the string motion will be effectively free, which means that the only significant dissipation mechanism will be that of radiation reaction. Moreover, during the first part of this period, in the range $T_\star > T > T_\sigma$, the only radiation mechanism will be gravitational, which is so weak that to begin with it will have no perceptible effect at all, so that there will be an interval during which the smoothing length ξ remains roughly constant at the value $\xi_\star$ it attained at the end of the friction-dominated era, which will be given, according to (32) and (33) by

$$\xi_\star \approx \frac{\beta^2 m_P^3}{T_x^4}. \quad (43)$$

The effect of string intersections might even cause the effective smoothing length to creep downwards slightly, but there is another weak effect with an opposite tendency. In an accurate analysis the global damping mechanism of the cosmological expansion needs to be taken into account, whose effect is closely analogous to that of friction considered in the preceding regime, though with a damping time scale τ that is of the same order of magnitude as the Hubble time scale τ, so that the effect of this ''Hubble damping'' is only marginal.

During the last stage before the protovortons are formed, in the range $T_\sigma > T > T_f$, there will already be currents on the strings, which means that if they are electromagnetically coupled (as is often taken for granted though it is not necessarily the case) then the mechanism of gravitational radiation damping may in principle be reinforced by the potentially much stronger mechanism of electromagnetic radiation damping. However in practice even in the coupled case, the expected currents will be too weak for this be important, so throughout the range $T_\star > T > T_f$ the gravitational radiation is the only kind that actually matters.

The resulting gravitational smoothing scale ξ will be the length of the shortest loop for which the cosmological time scale (9) is exceeded by the relevant radiation survival time scale, for which the usual order of magnitude estimate has the form

$$t \approx \frac{m_P^2 \xi}{\Gamma G m_x^2}, \tag{44}$$

where Γ is a dimensionless coefficient of order unity, and where, for the heavy GUT strings—the kind most commonly considered in cosmology—the gravitational factor will be given by $(m_x/m_P)^2 \approx 10^{-6}$. The validity of the formula (44) has been confirmed in many particular cases by numerical simulations [8], though the value of the coefficient turns out to be typically

$$\Gamma \approx 10^2. \tag{45}$$

Equating (44) to the cosmological time scale (9), the corresponding cutoff length scale can be estimated as

$$\xi \approx \frac{\Gamma}{\sqrt{g^*} m_P} \left(\frac{T_x}{T} \right)^2. \tag{46}$$

This formula (46) will determine the scale of the smallest surviving structure after the temperature has fallen below a critical value $T_\dagger < T_\star$ at which this value of ξ becomes larger than the value (43). It can be seen from (46) that this value will be given by

$$T_\dagger \approx \left(\frac{\Gamma}{\sqrt{g^*}} \right)^{1/2} \frac{T_x^3}{\beta m_P^2}. \tag{47}$$

The relation (46) is equivalently expressible in the form

$$\xi \approx \kappa t, \tag{48}$$

where κ is a constant given by

$$\kappa \approx \Gamma \left(\frac{T_x}{m_P} \right)^2, \tag{49}$$

which in the case of GUT strings means $\kappa \approx 10^{-4}$.

As was discussed in more detail in the preceeding work [5], formulas ressembling (48) have been commonly employed in published discussions of string simulations [8], but there is some confusion arising from the use of various definitions of κ which lead to correspondingly diverse numerical values, usually larger than (49) for the practical reason that the simulations in question have limited resolution and in any case neglect the gravitational reaction mechanism from which (49) is derived. To avoid ambiguity we can of course simply use the formula (48) as a defining relation to specify the parameter κ during the Hubble damping ''doldrum'' regime $T_\star \gtrsim T \gtrsim T_\dagger$ (to which most of the simulations are restricted), that is to say, after friction has become negligible but before radiation reaction has had time to be effective, but with this convention κ will have to be considered as a function of time, starting with unit value $\kappa \approx 1$ at the begining of the ''doldrum'' regime and decreasing to the very low value (49) at which it levels off after the end of this transition regime at $T \approx T_\dagger$.

Insofar as the quantity ν in (11) is concerned, it can be expected that, although it will have a more complicated transitional behavior near the lower cutoff value $R/t \approx \kappa$, where κ is the gravitational damping constant given by (49), it is resonable to expect that in the intermediate range $\kappa \ll R/t \ll 1$ it should be given by a simple power scaling law of the form

$$\nu \approx \nu_\star \left(\frac{R}{t} \right)^\zeta, \tag{50}$$

for constant values of the index ζ and the coeficient $\nu_\star$, where the latter must necessarily be identified with the order of unity constant $\nu_\star$ that characterized the earlier, friction-dominated regime, in order to ensure continuous matching at the horizon scale, $R/t \approx 1$, above which the Brownian description will still prevail. It is shown in [11] that in the radiation-dominated regime with which we are concerned here there are reasons to expect that the appropriate value of the index should be close to but perhaps slightly greater than a lower limit given by

$$\zeta = \frac{3}{2}. \tag{51}$$

(The analogue for the matter era in which we are situated today is a value slightly greater than a lower limit given by $\zeta = 2$.)

Assuming that the formula (50) still gives the right order of magnitude at the lower end of its range (where for higher accuracy a less simple fomula would be needed) the averaged value $\overline{\nu}$ in the formula (13) for the number density n of small autonomous loops will be given in the radiation damping dominated regime $T \lesssim T_\dagger$ by the constant value

$$\overline{\nu} \approx \nu_\star \kappa^\zeta, \tag{52}$$

with κ given by (49), which means that the corresponding value of the loop number density itself will be given according to (48) by

$$n \approx \nu_\star \kappa^{\zeta - 3} t^{-3}. \tag{53}$$

Having thus obtained a reasonably plausible provisional estimate for the number density, the only thing that remains to be done to obtain all the elements wanted for working out the required result (29) is to obtain the value T_f of T at which the protovorton formation actually takes place. To do this we have to solve the equation (24) that results from the minimum length requirement, which can be seen from (46) to reduce to the simple form

$$\mathcal{Z}_f T_f^2 \approx \frac{\kappa m_P}{\sqrt{g^*}} \frac{T_\sigma^{i+1}}{m_x^i}. \tag{54}$$

However, before we can solve this deceptively simple equation in practice, we need to know the T dependence of the factor $\mathcal{Z}$. According to the reasoning described in the preceeding work [5], this factor can be expected to be given by an expression of the form

$$\mathcal{Z} \approx \left(\frac{T}{T_\sigma} \right)^{1-\varepsilon}, \tag{55}$$

where ε is a dimensionless loop production efficiency factor somewhere in the range $0 < \varepsilon \lesssim 1$. It is to be remarked that if the efficiency ε of loop production were zero, this would mean that the carrier field would be blueshifted by a factor that would be just the inverse of that by which the background radiation is redshifted. In practice, the fairly high loop production that is expected in the radiation damping regime implies that although there will still be a blueshift it will only be by a much more moderate factor.

The formula (55) provides what we need in principle to solve the equation (54) to obtain the value T_f of T at which the protovorton loops in which we are interested actually form. In practice, assuming (as is necessary for the scaling hypothesis to be justified) that there is no major phase transition significantly affecting the particle number weighting factor g^* characterizing the cosmological background in the temperature range under consideration—so that it can be taken to have the fixed value g^*_σ—the solution is conveniently obtainable in the explicit form

$$\frac{T_f}{T_\sigma} = \left[\frac{\kappa m_P}{\sqrt{g^*_\sigma} T_\sigma} \left(\frac{T_\sigma}{m_x} \right)^i \right]^{1/(3-\varepsilon)}. \tag{56}$$

For the resulting distribution of vortons, typically having the minimum size compatible with the criterion (8) which gives

$$E_v \approx m_x^2 / T_\sigma, \tag{57}$$

the estimates (46) and (52) can be used in conjunction with (56) to evaluate the formula (29) so that the mass density of the vorton distribution is finally found to be given by

$$\rho_v \approx f \nu_f \left(\frac{\sqrt{g^*_\sigma} m_P}{\Gamma m_x} \right)^{2-\varepsilon/(3-\varepsilon)} \left(\frac{T_\sigma}{m_x} \right)^{(3i-\varepsilon)/(3-\varepsilon)} T_\sigma T^3 \tag{58}$$

or equivalently

$$\frac{\rho_v}{T^3} \approx \varepsilon g^* \nu_\star \Gamma^{-1/2} \left(\frac{\sqrt{g^*_\sigma} m_P}{\Gamma m_x} \right)^{-\varepsilon/(3-\varepsilon)} \times \left(\frac{T_\sigma}{m_x} \right)^{(3i-\varepsilon)/(3-\varepsilon)} \left(\frac{m_x T_\sigma}{m_P} \right). \tag{59}$$

For the strictly chiral case $i = 1$ this expression reduces to

$$\frac{\rho_v}{T^3} \approx \varepsilon g^* \nu_\star \Gamma^{-1/2} \left(\frac{\sqrt{g^*_\sigma} m_P}{\Gamma m_x} \right)^{-\varepsilon/(3-\varepsilon)} \left(\frac{T_\sigma^2}{m_P} \right). \tag{60}$$

It can be see that in comparison with the previously studied generic case, $i = 2$, the result for this strictly chiral case is augmented by a factor $(m_x / T_\sigma)^{3/(3-\varepsilon)}$.

It is to be remarked that whereas in the case of condensation in the friction-dominated epoch the augmentation is attributable to an increase in the typical mass (39) of the vortons, whose number density is not affected, on the other hand in the case of condensation in the radiation-reaction-dominated regime the augmentation is attributable to an increase in the number density of the vortons, whose typical mass (57) is unaffected.

IV. THE NUCLEOSYNTHESIS CONSTRAINT

One of the most robust predictions of the standard cosmological model is the abundance of the light elements that were fabricated during primordial nucleosynthesis that occurred when the background temperature had a value, T_N, say, that is given roughly by the energy needed for tunneling through the Coulomb barrier between two protons, by

$$T_N \approx e^4 m_P \approx 10^{-4}\ \text{GeV}. \tag{61}$$

An essential ingredient of nucleosynthesis calculations is the current expansion rate of the universe, as determined by the cosmological background density ρ, which, in the accepted picture, was at that time still strongly radiation dominated, so that it would have had an order of magnitude ρ_N given by

$$\rho_N \approx g^* T_N^4. \tag{62}$$

In order to preserve this well-established picture, it is necessary that the vorton distribution should satisfy $\rho_v \lesssim \rho_N$ when $T \approx T_N$, which for the case of carrier condensation in the friction-dominated epoch is expressible as the condition

$$\frac{\varepsilon \nu_\star \beta T_\sigma^3}{g^*_\sigma m_P m_x T_N} \lesssim 1. \tag{63}$$

The condition of condensation in the friction-dominated regime will automatically be satisfied in the important case for which the carrier condensation occurs immediately after the string forming phase transition itself, i.e., for which one has $T_\sigma \approx m_x$. In this case (63) gives

$$T_\sigma \lesssim \left(\frac{g^*_\sigma m_P T_N}{\varepsilon \nu_\star \beta} \right)^{1/2}. \tag{64}$$

Substituting the expression (61) for T_N into this, taking $g^*_\sigma \approx 10^2$ and assuming that the factor $(\varepsilon \nu_\star)$ and the drag factor β are of the order of unity yields the inequality

$$T_\sigma \lesssim e^2 (g^*_\sigma m_P m_p)^{1/2} \approx 10^8\ \text{GeV} \tag{65}$$

as the condition that must be satisfied by the formation temperature of *cosmic strings in which a strictly chiral current condenses immediately*, subject to the rather conservative assumption that the resulting vortons last for at least a few minutes. It is to be observed that this condition is only marginally more severe than what was obtained for the nonchiral generic case [5] and that while it rules out the formation of such strings during any conceivable GUT transition, it is consistent by a wide margin with their formation at temperatures close to that of the electroweak symmetry breaking transition.

If instead of supposing that the current condensation was immediate, we now suppose that the strings were formed at the GUT transition, i.e., $m_x \approx T_{\rm GUT} \approx 10^{16}\,{\rm GeV}$, but that the condensation temperature T_σ is very much lower than this, then the analogous limit is obtained for a value of T_σ that lies in the radiation-reaction-dominated range governed by (60) so that as the analogue of (63) we shall obtain

$$\varepsilon \nu_\star \Gamma^{-1/2} \left(\frac{\sqrt{g_\sigma^*} m_P}{\Gamma T_{\rm GUT}} \right)^{-\varepsilon/(3-\varepsilon)} \left(\frac{T_\sigma^2}{m_P T_N} \right) \lesssim 1. \tag{66}$$

To find the highest temperature at which GUT strings can become chirally conducting without violating the nucleosynthesis constraints, we now set T_x equal to $T_{\rm GUT}$ in (63). Since ε is small compared to 3 its effect in the index can be neglected. We thereby obtain

$$T_\sigma \lesssim \left[\left(\frac{\Gamma^{1/2} m_P T_N}{\varepsilon \nu_\star} \right) \left(\frac{\Gamma T_{\rm GUT}}{\sqrt{g_\sigma^*} m_P} \right)^{-\varepsilon/(3-\varepsilon)} \right]^{1/2} \approx 10^9\ {\rm GeV}, \tag{67}$$

where, in the last step, we have used the estimates $g_\sigma^* \approx \Gamma \approx 10^2$ and $T_{\rm GUT} \approx 10^{-3} m_P\,{\rm GeV}$, and have neglected the dependence on the rather uncertain but supposedly order of unity quantities $(\varepsilon \nu_\star)$ and β. This limit is considerably lower (by a factor of the order of a thousand) than the analogous limit [5] for the generic nonchiral case, and consistently with what was postulated in its derivation, it can be seen to be well within the radiation-reaction-dominated range.

The upshot is that subject again to the rather conservative assumption that the resulting vortons survive until the time of element formation, which occurs within only a few minutes, a theory in which GUT cosmic strings become chirally conducting above $10^9\,{\rm GeV}$ is inconsistent with the observational data.

V. THE DARK MATTER CONSTRAINT

We now consider the rather stronger constraints that can be obtained if at least a substantial fraction of the vortons are sufficiently stable to last until the present epoch. On the basis of the standard (Einstein) theory of gravity, it is generally accepted that the virial equilibrium of galaxies and particularly of clusters of galaxies requires the existence of a cosmological distribution of ''dark'' matter with density considerably in excess of the baryonic matter that is directly observed, mainly in the form of stars, with density $\rho_b \approx 10^{-31}\,{\rm gm/cm^3}$. On the other hand, on the same basis, it is also generally accepted that to be consistent with the formation of structures such as galaxies, starting from initially small inhomogeneities in an approximately homogeneous background, it is necessary that the total amount of this ''dark'' matter should not greatly exceed the critical closure density, namely,

$$\rho_c \approx 10^{-29}\,{\rm gm\,cm^{-3}} \tag{68}$$

(which is about 2×10^{-123} in dimensionless Planck units). Extrapolating back, as a function of the cosmological temperature T, this maximum cold dark matter contribution will scale like the entropy density so that it will be given by the expression

$$\rho_c \approx g^* m_c T^3, \tag{69}$$

where m_c is a constant mass factor. Since $g^* \approx 1$ at the present epoch, the required value of m_c (which is roughly interpretable as the critical mass per black body photon) can be estimated as

$$m_c \approx 10^{-28} m_P \approx 1\ {\rm eV}. \tag{70}$$

The constraint to which this dark matter limit leads is expressible as

$$\Omega_v \equiv \frac{\rho_v}{\rho_c} \lesssim 1. \tag{71}$$

In the case of vortons formed as a result of condensation during the friction damping regime, which by (33) requires

$$\beta m_P T_\sigma \gtrsim T_x^2, \tag{72}$$

the relevant estimate for the vortonic dark matter fraction is obtainable from (41) as

$$\Omega_v \approx \frac{\nu_\star f \beta T_\sigma^3}{m_P m_x m_c}. \tag{73}$$

In particular this formula applies to the case $T_\sigma \approx m_x$ in which the carrier condensation occurs very soon after the strings themselves are formed, for which one obtains

$$T_\sigma \lesssim \left(\frac{m_P m_c}{\nu_\star f} \right)^{1/2} \approx 10^5\ {\rm GeV}. \tag{74}$$

However, if we are concerned with the general category of strings formed at the GUT level, then we shall be obliged to consider the case of vortons formed as a result of condensation during the gravitational radiation damping regimes

$$T_\sigma \ll \frac{T_x^2}{\beta m_P}, \tag{75}$$

for which by (60), the relevant estimate for the vortonic dark matter fraction is obtained as

$$\Omega_v \sim \varepsilon g^* \nu_\star \Gamma^{-1/2} \left(\frac{\sqrt{g_\sigma^*} m_P}{\Gamma m_x} \right)^{-\varepsilon/(3-\varepsilon)} \left(\frac{T_\sigma^2}{m_P m_c} \right). \tag{76}$$

Setting T_x equal to $T_{\rm GUT}$ in this formula, and dropping the order of unity coefficients in view of the low indices involved, we obtain the corresponding limit as

$$T_\sigma \lesssim \left[m_P m_c \left(\frac{\Gamma T_{\rm GUT}}{\sqrt{g_\sigma^* } m_P} \right)^{-\varepsilon/(3-\varepsilon)} \right]^{1/2} \approx 10^5 \text{ GeV}, \quad (77)$$

where, as for the analogous nucleosynthesis limit (67), the last step is based on use of the estimates $g_\sigma^* \approx \Gamma \approx 10^2$ and $T_{\rm GUT} \approx 10^{-3} m_P$ GeV, and on neglect of the dependence on the rather uncertain but supposed order of unity quantities (ε and $\nu_\star$).

VI. CONCLUSIONS

In this paper we have constrained particle physics theories that give rise to chiral current-carrying strings. Such chiral strings arise in a class of supersymmetric theories where the symmetry is broken with a D term. Our bounds apply to all such theories, including those derived from superstring theories. We have derived these bounds from cosmological considerations, requiring that the resulting stable loops, or vortons, do not dominate the energy density of the universe. We have considered two possibilities. Either the vortons live only a few minutes, in which case we demanded that the universe be radiation dominated at the time of nucleosynthesis. If the vortons are stable enough to survive to the present time, then we demanded that they do not overclose the universe.

The especially simple dynamical behavior [7] of strictly chiral strings and the related consideration that their vorton states will have only half as many degrees of freedom of internal excitation, and therefore only half as many conceivable decay modes as in the case of generic nonchiral Witten currents means that these strictly chiral vortons can be expected to be very much more stable than most other kinds. It therefore seems highly plausible that they would survive not merely to the time of nucleosynthesis but to the present cosmological epoch, which means that it is the rather severe limits (74) and (77) that are relevant. In both of these cases the upper limit for the energy of the chiral current forming phase transition (whether the strings themselves were formed in the same transition or much earlier at the GUT transition) is found to be of the order of 10^5 GeV, which happens to be near the upper limit of the range considered likely to characterize the electroweak phase transition.

The preceding conclusion has both negative and positive implications. On the negative side it seems to exclude theories in which chiral string currents are formed at energies much larger than the electroweak level. On the positive side it suggests that vortons supported by strictly chiral currents that condensed during, or just above, the electroweak phase transition might conceivably form a significant fraction of the dark matter in the universe. Such considerations could well be relevant to superstring theories with large extra dimensions.

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